Coupling and higher-order effects in the ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions

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Coupled-channel calculations are performed for the ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions between 7 and 60 MeV to study the effect of inelastic couplings in transfer reactions. The effect of treating transfer beyond Born approximation is also addressed. The coupling to the ^{12}C 2^+ state is found to change the peak cross section by up to 15%. Effects beyond Born approximation lead to a significant renormalization of the cross sections, between 5% and 10% for deuteron energies above 10 MeV and larger than 10% for lower energies. We also performed calculations including the remnant term in the transfer operator, which has a small impact on the ^{12}C$(d,p)^{13}$C(g.s.) and ^{13}C$(p,d)^{12}$C(g.s.) reactions (where g.s. indicates ground state). Above 30-MeV deuteron energy, the effect of the remnant term is larger than 10% for the ^{12}C$(d,p)^{13}$C$(1/2^+, 3.09$ MeV) reaction and is found to increase with decreasing neutron separation energy for the 3.09-MeV state of ^{13}C. This is of importance for transfer reactions with weakly bound nuclei.

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I. INTRODUCTION

Single-nucleon transfer reactions that probe the degrees of freedom of single particles have been extensively used to study the structure of stable nuclei. The analysis of such reactions provides the angular momentum transfer [1], which gives information on the spin and parity of the final state. The sensitivity of the cross sections to the single-nucleon components allows for the extraction of spectroscopic factors. These observables quantify the overlaps between nuclear states and can be used to deduce occupancies of single-particle orbitals in nuclei [1]. Thanks to recent developments in radioactive beam production and in the detection of light charged particles, transfer reactions can now be used to investigate the single-particle structure of exotic nuclei. Experimental programs on transfer reactions with radioactive beams have indeed been initiated by several teams [2–4]. More recently, nuclear knockout reactions [5] were shown to be another useful tool to extract spectroscopic factors for loosely bound nuclei. The recent indications of reduced occupancies of single-particle states [5–7] reveal that reliable measurements of spectroscopic factors in exotic nuclei are highly desirable.

The analysis of transfer reactions most frequently relies on the distorted waves Born approximation (DWBA) [1]. This method presents uncertainties that are due to (i) the sensitivity to the optical-model parameters, (ii) the sensitivity to the single-particle parameters, and (iii) the assumptions behind the DWBA formalism itself. The sensitivity of DWBA cross sections to optical-model parameters has been the object of a recent investigation in the case of the ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions [8]. In that work, a reanalysis of existing data between 4- and 60-MeV deuteron energy was performed by use of the DWBA with consistent input parameters over the whole energy range. In particular, potentials for the deuteron and proton channels were obtained from global parametrizations. This was shown to reduce the variations of the measured spectroscopic factors as a function of incident energy. The smallest variations were obtained by use of adiabatic potentials for the deuteron channel, following the prescription by Johnson and Soper [9], to take deuteron breakup into account. With that procedure, the average spectroscopic factor for the ground-state to ground-state transition in ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions between 12 and 60 MeV was found to be 0.61 ± 0.09, the error being the rms variation over the energy [8]. This result is in excellent agreement with the prediction of the pioneering shell-model calculations of Cohen and Kurath for *p*-shell nuclei [10]. This success could be seen as an indication that calculations similar in form to the DWBA but using adiabatic deuteron potentials give a good description of the (d,p) and (p,d) reaction mechanisms. However, in [8] no direct checks of effects beyond these approximations were done.

The DWBA assumes that elastic scattering is the dominant process in the entrance and exit channels [1]. The relative motion of the reaction partners in each channel is thus approximated by a distorted wave describing elastic scattering. The transfer transition amplitude is assumed to be small enough to be computed in Born approximation [1]. In the case of the (d,p) and (p,d) reactions, the description of the data is generally improved when the effect of deuteron breakup is taken into account by use of an adiabatic deuteron potential [11,12], as already mentioned. Such an adiabatic potential is used in place of the optical potential to generate the distorted wave for the deuteron channel, whereas the rest of the calculation is the same as in the DWBA. This procedure is referred to as the adiabatic distorted wave(ADW) method in the following text. In that framework, the process is still assumed to be elastic with respect to the target.

This paper reports on coupled-channel calculations performed for the ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions to test the effect of target excitation in entrance and exit channels. The effect of treating transfer beyond Born approximation was also addressed. Usually, in the analysis of (d,p) and

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(p,d) reactions, the transfer operator is replaced with the neutron-proton interaction alone. We checked the effect of this approximation by performing calculations with a more complete transfer operator.

In most coupled-channel calculations, the optical potentials are adjusted so that the predicted elastic scattering computed with the couplings reproduces elastic-scattering data. This procedure is not applicable to the present work, because we used adiabatic deuteron potentials that are not meant to describe elastic scattering. We are interested in a systematic estimation of the effect of couplings and not in performing the most precise calculation for the transfer cross section at a given energy. The latter would require detailed comparisons between measured and calculated cross sections for all the included channels and fine adjustments of potentials and coupling strengths. The estimation of the magnitude of the errors due to the neglect of couplings in the ADW method, however, does not depend on such details. The main goal of our work is to produce such an estimation to provide guidelines for the analysis of transfer reactions with stable or weakly bound nuclei.

II. INELASTIC COUPLINGS

We performed coupled-channel calculations for the ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions, including transfer routes through the 4.44-MeV 2^+ state of ^{12}C and the 3.09-MeV $1/2^+$ state of ^{13}C. The calculations were performed with the code FRESCO [13] in the framework of the coupled-channel Born approximation (CCBA) [1]. We used adiabatic potentials for the deuteron channel; thus our treatment differs from usual CCBA calculations. We have shown that the adiabatic approximation for deuteron breakup can be introduced in the coupled-channel treatment of inelastic target excitations [14]. In the case of transfer, this leads to coupled equations and a transfer transition amplitude similar in form to that of the usual CCBA [14]. For the sake of clarity, our treatment is dubbed ACC (for adiabatic coupled channels) in the following text. In this framework, target excitations in the entrance and exit channels are included explicitly and treated to all orders by the solutions of coupled equations, whereas the transfer is still treated with Born approximation. The coupling scheme of Fig. 1 summarizes the transfer routes and excitations included in the calculations in the case of the ^{12}C$(d,p)^{13}$C reaction.

The 2^+ state of ^{12}C was assumed to be a collective excitation of rotational nature. We included the coupling between this state and the ^{12}C ground state by deforming the $d+^{12}$C potential. The deformation length was obtained from the adopted experimental value of $39.7 \pm 3.3\ e^2$ fm^4 for the $B(E2)$ of ^{12}C [15]. Using the collective model with a value of $1.25 \times 12^{1/3}$ fm for the radius of ^{12}C, we found the deformation length to be $\beta_2 R = -1.54$ fm. We choose the negative sign following experimental indications of an oblate deformation [16].

The $1/2^+$ first excited state of ^{13}C was assumed to be a single-particle $E1$ transition. We performed the single-neutron excitation in ^{13}C that was due to the interaction with the proton by using a folded coupling potential in place of the ^{13}C$+p$ optical potential. This coupling potential was taken as the sum of a ^{12}C$+p$ potential and a neutron-proton interaction. Details on the ^{12}C$+p$ potential are given below. The neutron-proton part was a Gaussian interaction, with parameters taken from [17] and reproducing the rms radius and binding energy of the deuteron.

Spectroscopic amplitudes for the overlaps of the ^{13}C states with the ^{12}C states were obtained from shell-model calculations, which we performed in the *psd* model space by using the code OXBASH [18], with the interaction named PSDMK in the OXBASH nomenclature. This interaction is composed of three parts: (i) the Cohen and Kurath (8–16)POT potential representation for the p shell [19], (ii) the *sd*-shell interaction of Preedom and Wildenthal [20], and (iii) the *p-sd* cross-shell interaction of Millener and Kurath [21]. The 0^+ ground state and 2^+ state of ^{12}C and the $1/2^-$ ground state of ^{13}C were assumed to be 0 $\hbar\omega$ states, whereas the $1/2^+$ state of ^{13}C was assumed to be a 1 $\hbar\omega$ state. The resulting spectroscopic amplitudes are summarized in Table I. As a check, we also computed the $B(E2)$ reduced transition probability by using the ^{12}C 0^+ and 2^+ shell-model wave functions. We obtained a value of 50.3 e^2 fm^4, close to the adopted experimental value. The sign of the computed $E2$ transition matrix element was also found to be in agreement with the sign of the deformation length of ^{12}C.

The single-neutron radial form factors were computed in a Woods-Saxon well with a standard geometry ($r = 1.25$ fm and $a = 0.65$ fm, as used in [8]). We calculated each form factor separately by adjusting the depth of the potential in order to obtain the experimental binding energy, and then we normalized by the relevant shell-model spectroscopic amplitude.

Apart from the aspects of the calculations specific to the coupled-channel approach, our work has a few minor differences with the work of Liu *et al*. [8]:

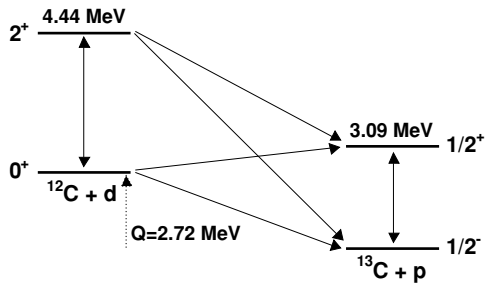

FIG. 1. Coupling scheme for the ^{12}C$(d,p)^{13}$C reaction.

TABLE I. Spectroscopic amplitudes for the $\langle^{12}\mathrm{C} \otimes (n\ell j)_\nu | ^{13}\mathrm{C}\rangle$ overlaps, as computed from the shell-model calculations described in the text.

^{12}C$(J^\pi) \otimes (n\ell j)_\nu$	^{13}C$(1/2^-$, g.s.)	^{13}C$(1/2^+$, 3.09 MeV)
$0^+_{\mathrm{g.s}} \otimes 1p_{1/2}$	−0.7755	
$2^+_1 \otimes 1p_{3/2}$	1.0592	
$0^+_{\mathrm{g.s}} \otimes 2s_{1/2}$		−0.9282
$2^+_1 \otimes 1d_{5/2}$		−0.3089

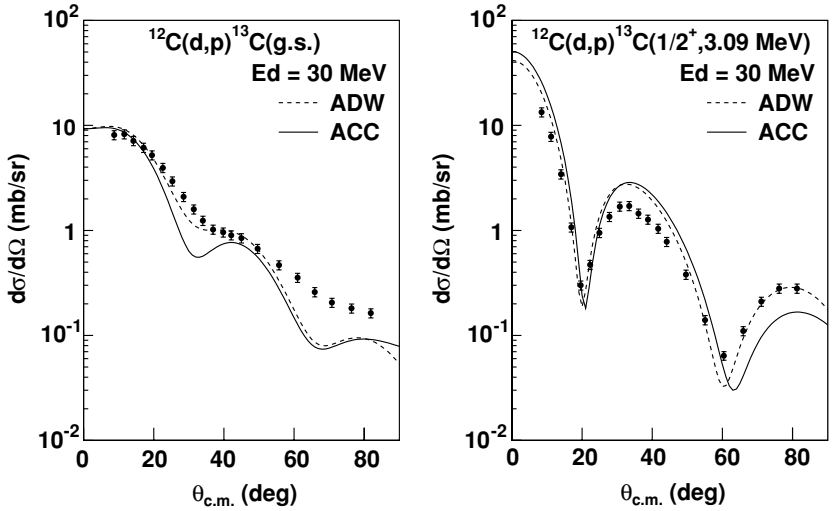

FIG. 2. Experimental, ADW and ACC angular distributions for the ^{12}C$(d,p)^{13}$C reaction leading to the ground state and first excited state of ^{13}C, at 30-MeV incident energy.

(1) We performed our calculations in exact finite range, using the Reid soft core interaction for the neutron-proton interaction to compute the deuteron ground-state wave function and in the transfer operator. The representation of the transition amplitude [1] was chosen so as to obtain the neutron-proton interaction as the main transfer operator. We thus adopted the post representation for ^{12}C$(d,p)^{13}$C and the prior representation for ^{13}C$(p,d)^{12}$C.
(2) We included a spin-orbit interaction in the single-particle potential used to compute the neutron form factors for ^{13}C, with a depth of $V_{SO} = 6$ MeV .
(3) We did not include any nonlocality corrections in the calculations, as these corrections are very complicated within the coupled-channel formalism.

These modifications are expected to have the same effect on ADW and ACC calculations and therefore should not change our conclusions on couplings. All other aspects of the calculations were chosen to be the same as those in the study by Liu *et al*. [8]. We therefore used the same adiabatic potentials for the deuteron channels. The $n+^{12}$C and $p+^{12}$C optical potentials used to build these potentials as well as the $p+^{13}$C optical potentials were obtained from the Jeukenne-Lejeune-Mahaux (JLM) microscopic nucleon-nucleon interaction [22]. Further details on these potentials can be found in the paper by Liu *et al*. [8]. The ^{12}C$+p$ part of the ^{13}C$+p$ coupling potential used in the ACC calculations including the excitation of the ^{13}C 3.09-MeV state was also obtained using the JLM interaction.

In Fig. 2, we present results of ADW and ACC calculations for the ^{12}C$(d,p)^{13}$C reaction leading to the ground and first excited states of ^{13}C at 30-MeV deuteron energy, as compared with the data of Ohnuma *et al*. [23]. ADW calculations were performed with the same parameters as were used for ACC calculations, but without inelastic couplings in the entrance and exit channels. Therefore transfer paths through an excited state of ^{12}C and/or ^{13}C are absent from the ADW calculations. These calculations were not normalized to the data, and no attempt was made to improve the description of the data. Nevertheless, Fig. 2 shows that both ADW and ACC calculations can reproduce the magnitude of the forward angle cross sections for the ^{12}C$(d,p)^{13}$C reaction. When comparing ACC and ADW angular distributions, one sees that the couplings change both the shape and the amplitude in the peak region, where spectroscopic factors are usually extracted by normalization of calculated cross sections to the data. At 30 MeV, the change in the cross section at the angle corresponding to the ADW peak is -3% for the transition to the ground state of ^{13}C and +22% for the transition to the first excited state of ^{13}C.

From Fig. 2, it also appears that the description of the data is not improved when ACC calculations are done. Better agreement might be obtained if the deuteron potential were constrained to fit the deuteron elastic-scattering data. Here, the deuteron potential in the ACC calculations was kept the same as in the ADW calculations. The couplings to excited states modified the $d+^{12}$C(g.s.) and $p+^{13}$C(g.s.) relative wave functions which could have an impact on the transfer cross sections. In order to clarify this effect, we performed another series of ACC calculations with one-way couplings only, i.e., without back couplings from the excited states of the ^{12}C and ^{13}C to their respective ground states. This means that no couplings were taken into account in the coupled equations describing the relative motions of the ^{12}C(g.s.)+d and ^{13}C(g.s.)+p systems. Thus, in these latter calculations, the elastic wave functions were not modified in the entrance channel or in the exit channel when going from ADW to ACC. We found that the effects of one-way couplings are of the same magnitude and sign as those of the full couplings.

Describing deuteron elastic scattering and deuteron breakup at the same time would require treating the deuteron breakup in an explicit way by use of more complicated models, such as the continuum discretized coupled-channels (CDCC) method. Such calculations were performed recently by Keeley, Alamanos, and Lapoux [24] for the ^{12}C$(d,p)^{13}$C reaction at 15- and 30-MeV deuteron energy, also including the excitation of the ^{12}C 2^+ state. Although these authors used a more realistic description of the deuteron breakup through the CDCC method and used deuteron elastic-scattering data to adjust the $d+^{12}$C potentials, the quality of the agreement between the data and their calculations is at the same level as for our ACC calculations. It is not the purpose of this work to provide a

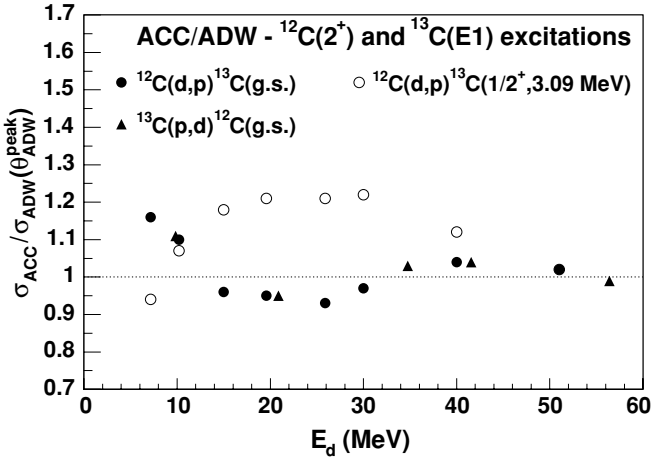

FIG. 3. Ratio of the ACC to ADW cross sections taken at the angle of the ADW peak for the ^{12}C(d,p) reaction leading to the ground state and first excited state of ^{13}C and for the ^{13}C(p,d) reaction leading to the ground state of ^{12}C. The ACC calculations include excitation of both the ^{12}C 2^+ and ^{13}C $1/2^+$ states.

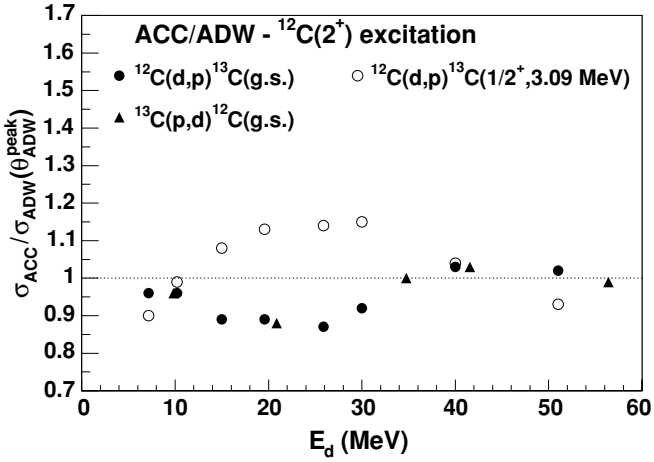

FIG. 4. Ratio of the ACC-to-ADW cross sections taken at the angle of the ADW peak for the ^{12}C(d,p) reaction leading to the ground state and first excited state of ^{13}C and for the ^{13}C(p,d) reaction leading to the ground state of ^{12}C. The ACC calculations include the excitation of the ^{12}C 2^+ state only and neglect the $E1$ coupling to the ^{13}C 3.09-MeV state.

good fit to the data but rather to probe whether the neglect of couplings to excited states in the ADW approach leads to a significant error in the extracted spectroscopic factors.

In Fig. 3, we show the ratio of ACC to ADW cross sections at the ADW peak as a function of deuteron energy for the ^{12}C$(d,p)^{13}$C reaction leading to the ground and first excited states of ^{13}C, and for the ^{13}C$(p,d)^{12}$C reaction leading to the ^{12}C ground-state. For the ground-state to ground-state transitions, the effect seems to be increasing with decreasing energy and shows an oscillatory behavior. We also note that couplings have identical effects on the ^{12}C$(d,p)^{13}$C(g.s.) and ^{13}C$(p,d)^{12}$C(g.s.) reactions, as expected from the detailed balance principle. For deuteron energies above 10 MeV, the effect on the ground-state to ground-state transitions is of the order of 5%. The effect is much stronger in the case of the ^{12}C$(d,p)^{13}$C$(1/2^+$, 3.09 MeV) reaction and leads to a change in the peak cross section larger than 15% for deuteron energies between 15 and 30 MeV. This shows that the effect of couplings can be large and depends on the final state and on the incident energy. In other words, a small coupling effect at a particular energy does not mean that the effect is small at all energies.

Previous CCBA analyses for the ^{12}C$(d,p)^{13}$C reaction to positive-parity states in ^{13}C were performed at 15-MeV [25] and 30-MeV [23] deuteron energy, with optical potentials for the deuteron channel. At 15 MeV, the effect on the $1/2^+$ state of ^{13}C that is due to the coupling to the ^{12}C 2^+ state is smaller that the one observed in the present work. This might be attributed to different choices for the deuteron+^{12}C potential. At 30 MeV, the effect from the 2^+ is comparable with our observations. Those two previous studies did not address the effect of couplings on the transfer to the ^{13}C ground state.

One can test the assumption of single-particle E1 excitation for the ^{13}C $1/2^+$ state at 3.09 MeV by comparing the experimental $B(E1)$ for the corresponding transition to the $B(E1)$ calculated with the neutron form factors and the shell-model spectroscopic amplitudes. The computed $B(E1)$ is 0.28 Weisskopf units (W.u.) whereas the experimental value is 0.047 ± 0.010 W. u. [26]. This strong overestimation is due to the fact that a significant fraction of the $E1$ strength is actually located at higher excitation energy, in the region of the giant dipole resonance, which was not taken into account here. Consequently, the effect of the excitation of the ^{13}C first excited state on the transfer is overestimated. To clarify the uncertainty concerning this E1 excitation, we performed ACC calculations with the excitation of the ^{12}C 2^+ state alone. The ratios of these ACC cross sections to the ADW cross sections are shown in Fig. 4. We see that the overall effect of the excitation of the ^{12}C 2^+ state is similar to the effect we obtained by including both the 2^+ and $E1$ excitations. Again, ACC-to-ADW ratios for the ^{12}C$(d,p)^{13}$C(g.s.) and ^{13}C$(p,d)^{12}$C(g.s.) reactions show identical behaviors, which implies that the coupling to the 2^+ state has the same effect whether it occurs in the entrance or exit channel. Between 15 and 30 MeV, switching off the $E1$ excitation changes the peak cross section by about 5%. Below 15 MeV, this change is of the order of 10%. For the ^{12}C$(d,p)^{13}$C$(1/2^+$, 3.09 MeV) reaction, the effect of removing the $E1$ excitation in ^{13}C amounts to a 5% change in the cross section at all energies. The excitation of the ^{12}C 2^+ state alone already has an important effect on transfer cross sections, which are changed by about 15% at 25 MeV for the three reactions studied. Because we used the experimental $B(E2)$ value to determine the intensity of the coupling between the ground state and the 2^+ state of ^{12}C, this excitation is not subject to the same uncertainty as the coupling to the ^{13}C 3.09-MeV state.

III. THE REMNANT TERM

In the post representation of the DWBA for the ^{12}C$(d,p)^{13}$C reaction, the transfer operator is [1]

$$W_{^{13}Cp} = V_{^{13}Cp} - U_{^{13}Cp}, \quad (1)$$

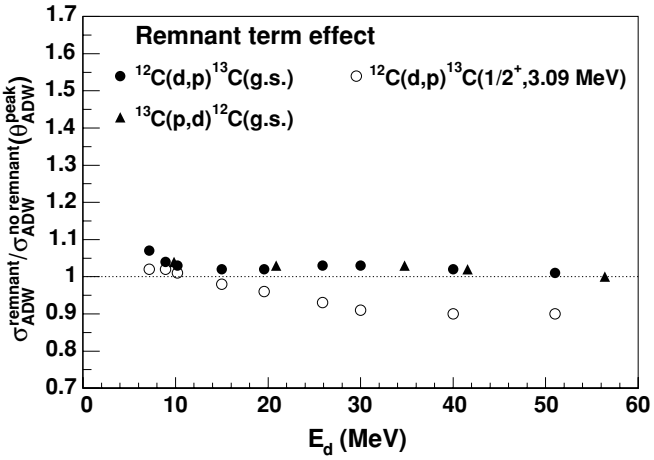

FIG. 5. Ratios of the ADW cross sections calculated with and without the remnant term as functions of deuteron energy.

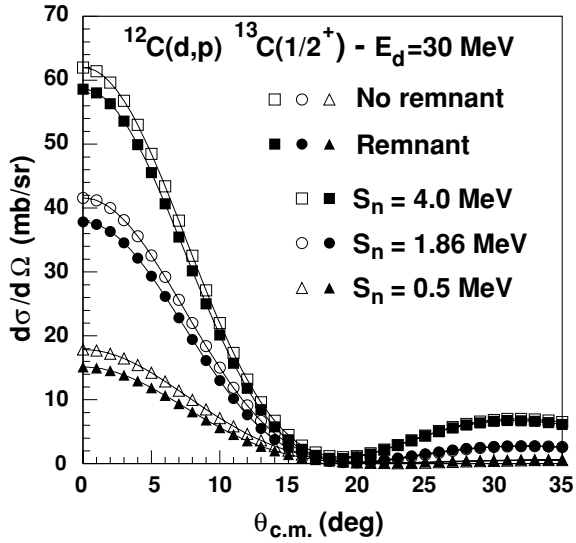

FIG. 6. ADW angular distributions for the ^{12}C$(d,p)^{13}$C$(1/2^+$, 3.09 MeV) reaction at 30 MeV, with and without the remnant term, and with different neutron separation energies for the final state. The curves are displayed to guide the eye.

where $V_{^{13}Cp}$ is the full effective interaction between the two nuclei in the exit channel, and $U_{^{13}Cp}$ is the optical potential used to generate the distorted wave in the exit channel. If one assumes that the full interaction is the sum of two-body interactions, $W_{^{13}Cp}$ can be decomposed into

$$W_{^{13}Cp} = V_{np} + V_{^{12}Cp} - U_{^{13}Cp}, \tag{2}$$

where $V_{^{12}Cp}$ is the *core-core interaction* and $V_{^{12}Cp} - U_{^{13}Cp}$ is called the *remnant term* or *indirect interaction*. The remnant term is usually neglected in DWBA and ADW calculations, leaving the neutron-proton interaction as the only transfer operator. This assumption is questionable, especially in the case of light and/or exotic nuclei for which the interaction V_{Ap} could differ significantly from the $V_{(A-1)p}$ interaction.

We performed a series of ADW calculations for the ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions, including the remnant term in the transfer operator. The core-core interactions were assumed to be complex, as indicated by Satchler [1], and were approximated by ^{12}C+p optical potentials obtained by use of the JLM interaction. In the transfer calculations, we included both real and imaginary parts of the remnant term. Ratios of ADW peak cross sections with and without the remnant term are displayed in Fig. 5 as functions of deuteron energy. Here again, effects on the ^{12}C$(d,p)^{13}$C(g.s.) and ^{13}C$(p,d)^{12}$C(g.s.) are found to be identical. The effect of the remnant term on these ground-state to ground-state transitions is smaller than 5%, except for the lowest energies, i.e., at about 8 MeV. In the case of the ^{12}C(d,p) reaction to the ^{13}C $1/2^+$ state at 3.09 MeV, inclusion of the remnant term lowers the peak cross section. This effect increases with energy and seems to saturate at 10% for energies over 30 MeV. Effects of the remnant term on ACC cross sections are very similar to the effects on ADW calculations.

The greater sensitivity of the ^{12}C$(d,p)^{13}$C$(1/2^+$, 3.09 MeV) reaction to the remnant term is particularly interesting. The neutron separation energy of the 3.09-MeV state in ^{13}C is only 1.86 MeV, whereas it is 4.95 MeV for the ground state. To test the relation between the separation energy and the effect of the remnant term, we performed ADW calculations for the ^{12}C$(d,p)^{13}$C reaction at 30 MeV, with and without the remnant term, and with neutron separation energies artificially modified for both states in ^{13}C. We calculated the transfer cross sections to a $1/2^+$ state in ^{13}C ($2s_{1/2}$ neutron configuration) with $S_n = 0.5$ and 4.0 MeV and to a $1/2^-$ state ($1p_{1/2}$ neutron configuration) with $S_n = 0.5$ and 2.0 MeV. The results for the transition to the $1/2^+$ state are shown in Fig. 6. The cross section decreases when the neutron separation energy decreases, whereas the absolute effect of the remnant term on the peak cross section is roughly constant. Therefore the relative effect of the remnant term increases when the neutron separation energy decreases. For a binding energy of 0.5 MeV, inclusion of the remnant term reduces the ADW peak cross section by 15%. For the transition to the $1/2^-$ state, the effect of the remnant term is smaller than 5% for $S_n = 4.95$ MeV and decreases with decreasing binding energy. At $S_n = 0.5$ MeV, the effect on the peak cross section is smaller than 1%. Clearly the effect of the remnant term depends on the properties of the populated neutron state. In particular, we find that large corrections are to be expected when transferring to loosely bound neutron $s_{1/2}$ orbitals, which play an important role in the appearance of nuclear halo states [27]. For the interaction of the proton and the ^{13}C, we used optical potentials built from the JLM interaction by using the ground-state ^{13}C matter density [8]. When more realistic optical potentials are used for loosely bound systems (typically more diffuse), the remnant term effect will be even larger than our estimate.

IV. COUPLED-REACTION-CHANNEL CALCULATIONS

If the transfer interaction is strong enough, it can induce multiple transfers between the initial and the final states, in the forward direction (i.e., from the entrance channel to the exit channel) and in the backward direction (i.e., from the exit channel to the entrance channel). One then

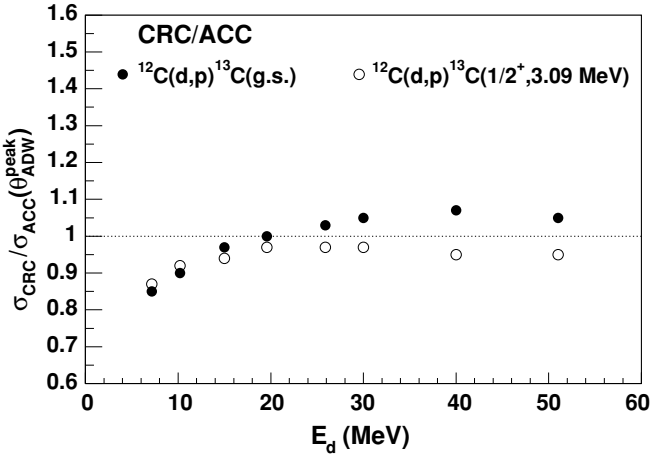

FIG. 7. Ratio of the CRC-to-ACC cross sections at the ADW peak angle for the ^{12}C$(d,p)^{13}$C(g.s.) and ^{12}C$(d,p)^{13}$C$(1/2^+$, 3.09 MeV) reactions.

needs to go beyond Born approximation to compute the transfer transition amplitude. To check the validity of Born approximation for the ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions, we performed coupled-reaction channel (CRC) calculations. In this method, the transfer is treated to an arbitrarily high order. This order is the number of times the nucleon is transferred between the entrance and exit partitions. When more than one transfer step is performed, one needs to take into account the nonorthogonality term [1] between the entrance and exit partitions. At each step in CRC, all couplings are treated by the solutions of coupled equations, as in the CCBA method, but also with a source term that is due to the transfer. The channel wave functions used in the source terms at a given step are taken as the results of the previous step. In the CRC calculations, we included the remnant term in the transfer operator, as well as the nonorthogonality correction. Results are shown in Fig. 7 in terms of the ratios of CRC and ACC cross sections as functions of deuteron energy. Convergence of the CRC results for the ^{12}C$(d,p)^{13}$C reaction was reached after eight transfer steps. In the case of the ^{13}C$(p,d)^{12}$C reaction, convergence was difficult to obtain because of the remnant term and/or the nonorthogonality correction. Therefore we do not show the corresponding results. Relative effects of the CRC on the ^{12}C$(d,p)^{13}$C(g.s) and ^{12}C$(d,p)^{13}$C$(1/2^+, 3.09$ MeV) reactions are similar to each other. Below 10-MeV deuteron energy, the CRC reduces the peak cross section by 10% or more. There is a global decrease with incident energy, reflecting the absolute transfer cross-section decrease with energy for both reactions. The transfer channel gets weaker with increasing energy, and effects beyond Born approximation tend to decrease. At 50-MeV deuteron energy, the effect saturates at about 5% for both reactions.

V. SUMMARY AND CONCLUSIONS

We performed coupled-channel calculations for the ^{12}C$(d,p)^{13}$C and ^{13}C$(p,d)^{12}$C reactions in order to test the effect of inelastic couplings. We studied the effect of the excitation of the 4.44-MeV 2^+ state in ^{12}C and of the ^{13}C $1/2^+$ state at 3.09 MeV. The coupling to the 2^+ state changes the peak cross section by up to 15%, which shows that attention should be paid to such excitations in transfer reactions. The rather strong energy dependence suggests that, when transfer reactions are analyzed, coupled-channel calculations performed to check inelastic couplings should be done at the energies of interest. Indeed, a small effect at a particular energy does not guarantee that the effect is small at all energies. Assuming that the $1/2^+$ state in ^{13}C is a single-particle $E1$ excitation, we found that the coupling to this state has a somewhat smaller effect on the transfer cross sections. Only at the lowest energies is this effect larger than 10%. Considering the fact that the $B(E1)$ reduced transition probability to this state is overestimated, the coupling effect would probably be significantly smaller when this excitation is taken into account in a more realistic way.

The effect of including the remnant term in the transfer operator for the ^{12}C$(d,p)^{13}$C(g.s.) and ^{13}C$(p,d)^{12}$C(g.s.) reactions is smaller than 5% for energies above 8 MeV. In the case of the ^{12}C$(d,p)^{13}$C$(1/2^+$, 3.09 MeV) reaction, the effect of the remnant term is found to be more important than for the ^{12}C$(d,p)^{13}$C(g.s.) reaction ($\approx$10%). We find that the relative effect of the remnant term on the transfer to the ^{13}C $1/2^+$ state increases with decreasing neutron separation energy, whereas the opposite happens for the ^{12}C$(d,p)^{13}$C$(1/2^-)$ transition. A large contribution from the remnant term should therefore be expected for neutron transfer reactions to or from loosely bound $s_{1/2}$ orbitals, e.g., neutron halo states.

We also addressed effects beyond Born approximation by performing CRC calculations, including the remnant term and the nonorthogonality correction. An effect larger than 10% is observed for deuteron energies below 10 MeV. Above that energy, changes in the peak cross sections are of the order of 5%. In the case of transfer reactions involving exotic nuclei, couplings to breakup states are expected to enhance the effect of going beyond Born approximation. However, nonorthogonality issues need to be clarified before a quantitative result is obtained. Work along these lines is planned for the ^{10}Be$(d,p)^{11}$Be reaction.

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