

Cross-sections of neutron rich nuclei from projectile fragmentation: canonical thermodynamic model estimates

G. Chaudhuri * † ¹, S. Das Gupta¹, W. G. Lynch², M. Mocko², and M. B. Tsang²

¹*Physics Department, McGill University, Montréal, Canada H3A 2T8 and*

²*National Superconducting Cyclotron Laboratory,*

Physics & Astronomy Department, Michigan State University,

East Lansing, Michigan 48824, USA

(Dated: July 5, 2007)

Abstract

A remarkably simple dependence of fragmentation cross-section on average binding energy has been established in experimental data. This dependence was empirically parametrised leading to a very useful formula for extrapolation. We find that the canonical thermodynamic model, which has been used in the past for successful computations of many observables resulting from multifragmentation, reproduces the salient features of fragmentation cross-sections of very neutron rich nuclei very well. This helps towards a theoretical understanding of the observed data.

PACS numbers: 25.70Mn, 25.70Pq

* gargi@physics.mcgill.ca

† On leave from Variable Energy Cyclotron Center, 1/AF Bidhan Nagar, Kolkata 700064, India

As part of a drive towards understanding the production mechanisms of rare isotopes, fragmentation cross-sections of many neutron rich isotopes have recently been measured from the ^{48}Ca and ^{64}Ni beams at 140 MeV per nucleon on ^9Be and ^{181}Ta targets [1]. Copper isotope cross-sections have been measured in projectile fragmentation of ^{86}Kr at 64 MeV per nucleon [2]. A remarkable feature is the correlation between the measured fragment cross-section and the average binding energy (Fig.1)[3]. This observation has prompted attempts of paramtrisation of cross-sections. One very successful parametrisation[3] is

$$\sigma = C \exp(A^{-1}(B - \varepsilon_{pair})/\tau) \quad (1)$$

Here B is the binding energy of the nucleus with mass number A , $\varepsilon_{pair} = \kappa\varepsilon A^{-3/4}$ and κ is 1 for even-even nuclei, 0 for odd-even nuclei and -1 for odd-odd nuclei. The pairing term smooths the straggling seen in the data when logarithms of cross-sections are plotted against B/A for even-even and odd nuclei (or odd-even and odd-odd nuclei). Theoretical basis for the simple appearance of B/A or the pairing term correction is not transparent. Another equation which is highly successful is

$$\sigma = cA^{3/2} \exp[(N\mu_n + Z\mu_p + B - \varepsilon_{pair} + E_s \cdot \min(S_n, S_p, S_\alpha))/T] \quad (2)$$

For values of parameters and details see [4].

Here we do calculations for the production cross-sections of silicon isotopes from projectile fragmentation of ^{48}Ca and of copper isotopes from projectile fragmentation of ^{86}Kr using the canonical thermodynamic model. Some of these cross-sections are very small and they serve as very stringent tests of the model. The model has been extensively applied for production cross-sections of other particles which are more numerous [5] and agreements are good. The basic physics of the model is the same as in many other models of intermediate energy heavy ion collisions: the statistical multifragmentation model (SMM) [6] or the microcanonical simulations of heavy-ion collisions [7, 8]. But SMM or the microcanonical simulations are totally impractical for calculations of very small cross-sections as they rely on Monte-Carlo simulations. The canonical model gives closed expressions and calculations can be made as accurate as desired. The grand canonical model is unsuitable for exploring these furthest limits of the phase-space and is expected to be very unreliable [5, 9]. We will come back to this point later.

We will consider production of silicon isotopes from the statistical break up of $^{48}_{20}\text{Ca}$. We denote the average number (multiplicity) of $^{14}\text{Si}_n$ by $\langle n_{14,n} \rangle$. Then the cross-section will be

$\sigma(14, n) = C \langle n_{14, n} \rangle$ where C is a constant not calculable from the thermodynamic model. It depends upon dynamics which is outside the scope of the model. Similarly we will consider the production of Cu isotopes ${}_{29}\text{Cu}_n$ from a source which has 36 protons and 50 neutrons, i.e., ${}^{86}\text{Kr}$. The source sizes adopted for the calculation are zero order guesses. It could be sometimes smaller or greater depending on the diffusion from the target.

We will write down the formulae used for the calculations but we will not derive them as they can be found elsewhere [5, 10]. The fragmenting system (${}^{48}\text{Ca}$ or ${}^{86}\text{Kr}$) has Z_0 protons and N_0 neutrons. The canonical partition function is given by

$$Q_{Z_0, N_0} = \sum \prod \frac{\omega_{i,j}^{n_{i,j}}}{n_{i,j}!} \quad (3)$$

Here the sum is over all possible channels of break-up (the number of such channels is enormous) which satisfy $Z_0 = \sum i \times n_{i,j}$ and $N_0 = \sum j \times n_{i,j}$; $\omega_{i,j}$ is the partition function of one composite with proton number i and neutron number j respectively and $n_{i,j}$ is the number of this composite in the given channel. The one-body partition function $\omega_{i,j}$ is a product of two parts: one arising from the translational motion of the composite and another from the intrinsic partition function of the composite:

$$\omega_{i,j} = \frac{V_f}{h^3} (2\pi m(i+j)T)^{3/2} \times z_{i,j}(\text{int}) \quad (4)$$

Here $m(i+j)$ is the mass of the composite and V_f is the volume available for translational motion; V_f will be less than V , the volume to which the system has expanded at break up. We use $V_f = V - V_0$, where V_0 is the normal volume of nucleus with Z_0 protons and N_0 neutrons. In this calculation we have used a fairly typical value $V = 6V_0$.

The probability of a given channel $P(\vec{n}_{i,j}) \equiv P(n_{0,1}, n_{1,0}, n_{1,1}, \dots, n_{i,j}, \dots)$ is given by

$$P(\vec{n}_{i,j}) = \frac{1}{Q_{Z_0, N_0}} \prod \frac{\omega_{i,j}^{n_{i,j}}}{n_{i,j}!} \quad (5)$$

The average number of composites with i protons and j neutrons is seen easily from the above equation to be

$$\langle n_{i,j} \rangle = \omega_{i,j} \frac{Q_{Z_0-i, N_0-j}}{Q_{Z_0, N_0}} \quad (6)$$

The constraints $Z_0 = \sum i \times n_{i,j}$ and $N_0 = \sum j \times n_{i,j}$ can be used to obtain different looking but equivalent recursion relations for partition functions. For example

$$Q_{Z_0, N_0} = \frac{1}{Z_0} \sum_{i,j} i \omega_{i,j} Q_{Z_0-i, N_0-j} \quad (7)$$

These recursion relations allow one to calculate Q_{Z_0, N_0}

We list now the properties of the composites used in this work. The proton and the neutron are fundamental building blocks thus $z_{1,0}(int) = z_{0,1}(int) = 2$ where 2 takes care of the spin degeneracy. For deuteron, triton, ${}^3\text{He}$ and ${}^4\text{He}$ we use $z_{i,j}(int) = (2s_{i,j} + 1) \exp(-\beta e_{i,j}(gr))$ where $\beta = 1/T$, $e_{i,j}(gr)$ is the ground state energy of the composite and $(2s_{i,j} + 1)$ is the experimental spin degeneracy of the ground state. Excited states for these very low mass nuclei are not included. For Si and Cu nuclei whose production cross-sections are sought in this work we use the experimental binding energies tabulated in [11] but also include a term for contribution from excited states (see the discussion following). For mass number $A = 5$ and greater (but charge $\neq 14(29)$) we use the liquid-drop formula. For nuclei in isolation, this reads ($a = i + j$)

$$z_{i,j}(int) = \exp \frac{1}{T} [W_0 a - \sigma(T) a^{2/3} - \kappa \frac{i^2}{a^{1/3}} - s \frac{(i-j)^2}{a} + \frac{T^2 a}{\epsilon_0}] \quad (8)$$

The derivation of this equation is given in several places [5, 6] so we will not repeat the arguments here. The expression includes the volume energy, the temperature dependent surface energy, the Coulomb energy and the symmetry energy. The term $\frac{T^2 a}{\epsilon_0}$ represents contribution from excited states since the composites are at a non-zero temperature. This form was used in other applications of the model and we have kept this unchanged. This term is also included in $z_{14,n}(int)(z_{29,n}(int))$. Note that in the fitting formula of Eq.(2) a different expression for contribution from excited states is used.

We have to state which nuclei are included in computing Q_{Z_0, N_0} . For i, j , (the proton and the neutron number) we include a ridge along the line of stability. The liquid-drop formula above also gives neutron and proton drip lines and the results shown here include all nuclei within the boundaries.

The long range Coulomb interaction between different composites can be included in an approximation called the Wigner-Seitz approximation. We incorporate this following the scheme set up in [6].

It remains now to state the results. Fig.1 taken from [3] shows the remarkable correlation between experimental values of cross-sections of silicon isotopes (from ${}^{48}\text{Ca}$ on ${}^9\text{Be}$ reaction at 140 MeV per nucleon) and average binding energies. The experimental data on cross-sections (shown as solid symbols in this paper) span about seven orders of magnitude. In Fig.2 we show results of our calculations(crosses). There are basically two parameters: an overall

normalisation factor (chosen in the figure to give the correct value of cross-section for ^{28}Si) and the temperature (taken here to be 9.5 MeV which is within the range of temperatures expected for this reaction). Except at the tails of the distribution, the agreement is fair and the calculation does indeed give the very rapid decrease of the cross-section for large A . The straggling in values of the cross-sections between even-even and odd nuclei is also reflected in the calculation. In Fig.3 we compare data and calculations for the case of production of copper isotopes. The data here span more than eight decades and the calculation, except for the tails, does very well. The straggling between cross-section values for odd-odd and odd copper isotopes is highlighted in Fig.4. In the same figure we show that both for data and calculation the straggling disappears if the cross-section is plotted against $\langle B \rangle - \varepsilon_{pair}$ rather than against just $\langle B \rangle$ (see also [4]). We find it gratifying that the model is able to reproduce such fine details.

Lastly, we will make a connection with grand canonical fitting of the data [4]. In our model we use $\sigma(z, n) = C \langle n_{z,n} \rangle$ where the value of C has to be taken from experiment. Thus in a model of this type what we need is the ratio $\langle n_{z,n+1} \rangle / \langle n_{z,n} \rangle$ to be predicted correctly. From Eq.(6) this is

$$\frac{\langle n_{z,n+1} \rangle}{\langle n_{z,n} \rangle} = \frac{\omega_{z,n+1}}{\omega_{z,n}} \frac{Q_{Z_0-z, N_0-n-1}}{Q_{Z_0-z, N_0-n}} \quad (9)$$

Here Z_0, N_0 are the charge and neutron number of the projectile which is fragmenting (^{86}Kr or ^{48}Ca) and (z, n) denotes Cu or Si isotopes. The right hand side of eq.(9) is very simple in the grand canonical ensemble. The ratio of the two canonical partition functions is replaced by a term independent of n , i.e., the right hand side is simply $\frac{\omega_{z,n+1}}{\omega_{z,n}} \exp(\beta\mu_n)$ where μ_n is a constant for all n, z . That would be correct if Z_0 and N_0 are large and also $Z_0 \gg z$ and $N_0 \gg n$ whereas in small systems such as here, the ratio of the partition functions varies as n changes. As we approach neutron drip line it drops fast with n . In order to approximate the right hand side of eq.(9) by the simpler expression $\frac{\omega_{z,n+1}}{\omega_{z,n}} \exp(\beta\mu_n)$ we need to choose μ_n judiciously and further alter the value of the temperature from the one used in the canonical model. In that case the temperature has to be significantly reduced.

In conclusion, the canonical model reproduces the salient features of production cross-sections of very neutron rich nuclei. The empirical formulae for extrapolation [3, 4] are very useful and the canonical thermodynamic model can not hope to replace these but it aids to a theoretical understanding of the data. The same parameters that we have used here

can be used to predict the production cross-sections of intrermediate mass fragments or the properties of the largest fragment after mutifragmentation [12].

I. ACKNOWLEDGEMENT

This work is supported by the Natural Sciences and Engineering Research Council of Canada and by the National Science Foundation under Grant No PHY-0606007.

-
- [1] M. Mocko et al., Phys. Rev. **C74**, 054612 (2006)
 - [2] M. Mocko et al., Phys. Rev. C (in press).
 - [3] M. Mocko et al., to be published in European Journal of Physics.
 - [4] M. B. Tsang et al., MSU-NSCL preprint 1350 (2007)
 - [5] C. B. Das, S. Das Gupta, W. G. Lynch, A. Z. Mekjian, and M. B. Tsang, Phys. Rep. **406**, 1 (2005).
 - [6] J. P. Bondorf, A. S. Botvina, A. S. Iljinov, I. N. Mishustin and K. Sneppen, Phys. Rep. **257**, 133 (1995).
 - [7] D. H. Gross, Phys. Rep. **279**, 119 (1997)
 - [8] J. Randrup and S. E. Koonin, Nucl. Phys. A **471**, 355c (1987).
 - [9] C. B. Das, S. Das Gupta, and B. K. Jennings, Phys. Rev **C70**, 044611 (2004)
 - [10] P. Bhattacharyya, S. Das Gupta and A. Z. Mekjian, Phys. Rev. **C60**, 054616 (1999)
 - [11] G. Audi, A. H. Wapstra, and C. Thibault, Nucl. Phys. A729, 337 (2003)
 - [12] G. Chaudhuri and S. Das Gupta, Phys. Rev. **C75**, 034603 (2007)

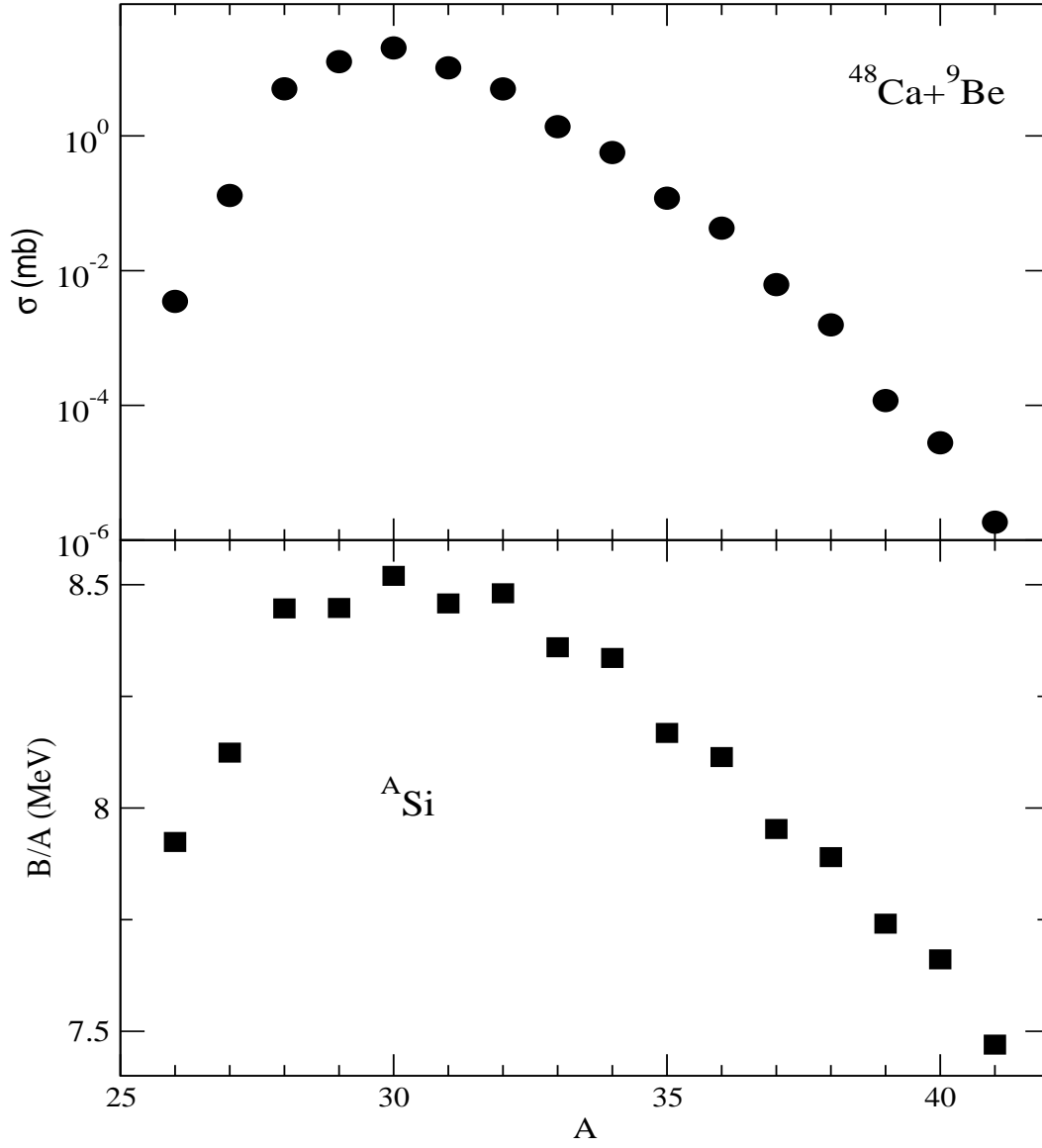


FIG. 1: Fragment cross-section and average binding energy plotted as a function of mass number for silicon isotopes.

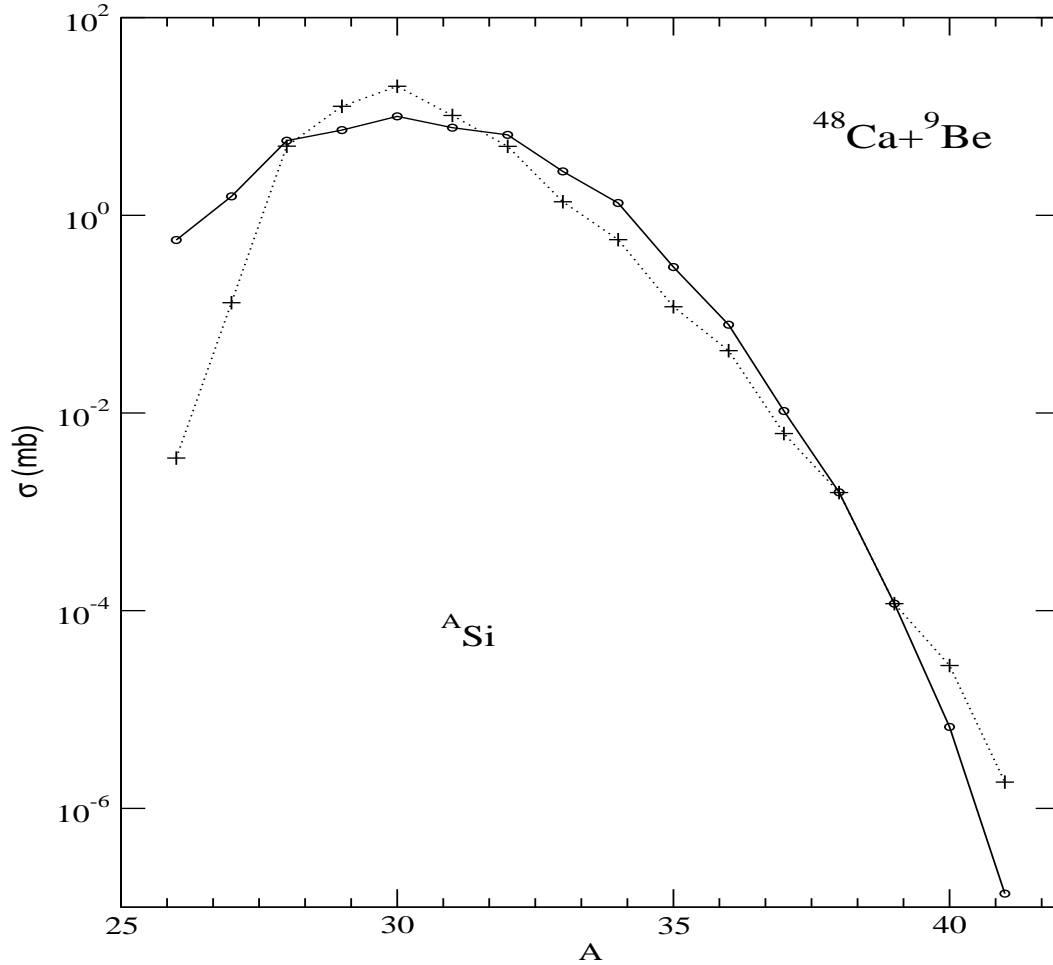


FIG. 2: Experimental cross-sections for silicon isotopes (points) compared with theoretical results (crosses). A dotted line is drawn through the experimental data and a solid line through the calculated values. The temperature used is 9.5 MeV. Normalisation constant for theory is chosen by fitting to the experimental cross-section of ^{39}Si

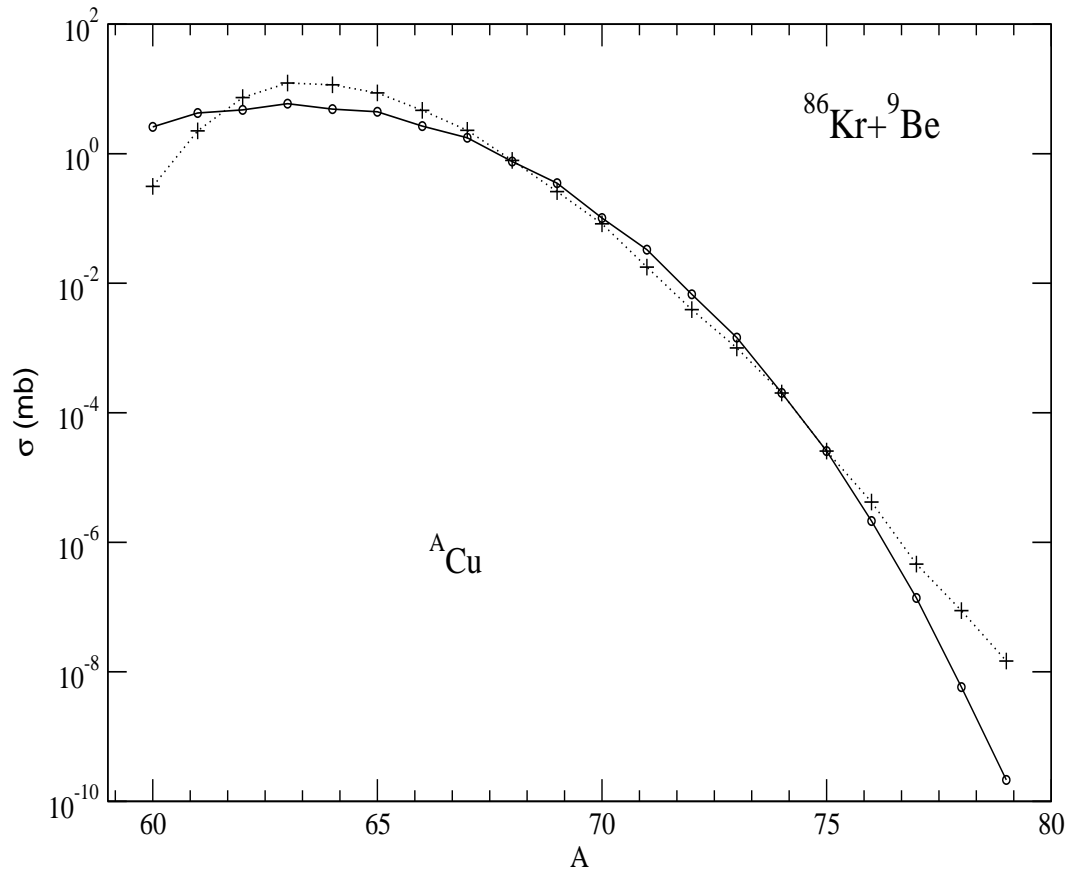


FIG. 3: Same as in Fig.2 but for Cu isotopes. Normalisation for theory is chosen from experimental value for ^{75}Cu .

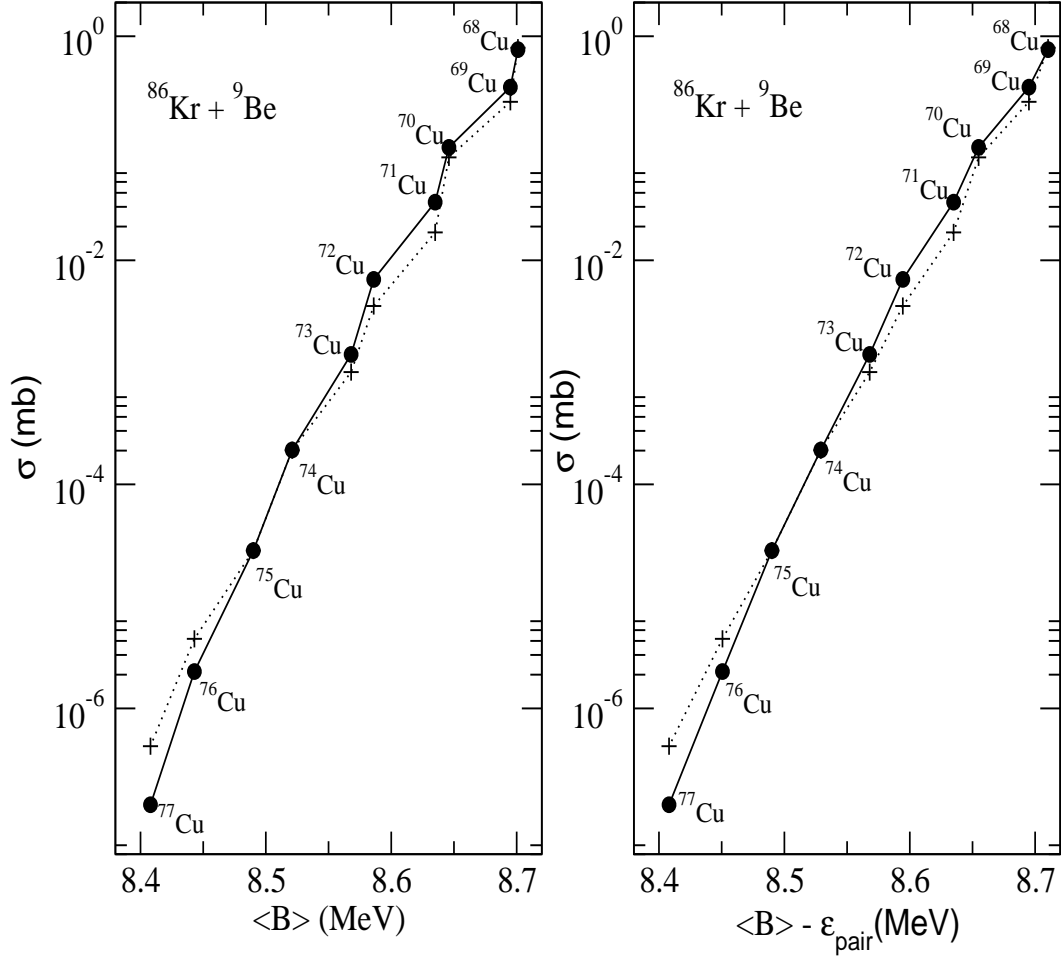


FIG. 4: Straggling of data between odd-odd and odd-even cases (points) when plotted against the average binding energy $\langle B \rangle = B/A$. Similar scatter is seen in theoretical calculation. On the right panel the cross-sections are plotted against $\langle B \rangle - \epsilon_{\text{pair}}$ (see eq.(1)). The value of ϵ used here is 30 MeV. This decreases the straggling significantly both for data and theory.