Exploring symmetry energy with emitted neutrons and protons using the Quantum Molecular Dynamics Model

Y.X. Zhang[1,2,3], P. Danielewicz[1,2,4], M. Famiano[5], Z. Li[3], W.G. Lynch[1,2,4], M.B. Tsang[1,2,4]

[1]*Joint Institute of Nuclear Astrophysics, Michigan State University, East Lansing, MI 48824, USA,*
[2]*National Superconducting Cyclotron Laboratory, Michigan State University, East Lansing, MI 48824, USA,*
[3]*China Institute of Atomic Energy, P.O. Box 275 (18), Beijing 102413, P.R. China,*
[4]*Physics and Astronomy Department, Michigan State University, East Lansing, MI 48824, USA.*
[5]*Physics Department, Western Michigan University, Kalamazoo, MI, USA.*

Abstract

Emissions of free neutrons and protons from the central collisions of ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn reactions are simulated using the Improved Quantum Molecular Dynamics model with two different density dependence of the symmetry energy in the nuclear equation of state. The constructed double ratios of the neutron to proton ratios of the two reaction systems are found to be sensitive to the symmetry terms in the EOS. The effect of cluster formation is examined and found to affect the double ratios mainly in the low energy region. In order to extract better information on symmetry energy with transport models, it is therefore important to have accurate data in the high energy region which also is affected minimally by sequential decays.

Information about the Equation of State (EOS) of asymmetric matter improves our understanding of the properties of neutron star such as stellar radii and moments of inertia, maximum masses [1-3], crustal vibration frequencies [4], and neutron star cooling rates [3,5], which are currently being investigated with ground-based and satellite observations. Recent observations of neutron stars with the XMM-Newton X-ray telescope have been interpreted as requiring an unusually repulsive equation of state for neutron matter [6]. It is important to determine whether such interpretations are supported by laboratory measurements. Measurements of isoscalar collective vibrations, collective flow and kaon production in energetic nucleus-nucleus collisions have constrained the equation of state for symmetric matter for densities ranging from normal saturation density to five times saturation density [7-9]. On the other hand, the extrapolation of the EOS to neutron–rich matter depends on the density dependence of the nuclear symmetry energy, which has comparatively few experimental constraints [10].

In the past decade, different probes in reaction experiments have been found to be sensitive to the symmetry energy term in the equation of state. These probes exploit the difference in the interactions of protons and neutrons either as free nucleons or as bound isotopes. They include isoscaling [11-13], isospin diffusion [14], neutron to proton (n/p) ratios ($R_{n/p}$) [15,16], neutron and proton flow [17], π^+/π^- ratios and π^+/π^- flow [18, 19]. In principle, the neutron to proton ratios [15, 20] have a more straight forward link to the symmetry energy than the other probes. However, experimentally, detection of neutrons is more difficult than detection of protons. Experimentally neutrons and protons are usually measured by two different detection systems. To remove the sensitivity to the different efficiencies in the proton and neutron detection systems, the double ratio DR(n/p)=$R_{n/p}(A)/R_{n/p}(B)$ is constructed from the experimental measurements.

$$R_{n/p}(A)/\, R_{n/p}(B) = \frac{dM_n(A)/dE_{c.m.}}{dM_p(A)/dE_{c.m.}} \cdot \frac{dM_p(B)/dE_{c.m.}}{dM_n(B)/dE_{c.m.}}$$

where $dM_n(A)/dE_{cm}$ and $dM_p(B)/dE_{cm}$ are energy spectra for neutrons and protons for two collision systems A and B which have different isospin contents.

The sensitivity of this observable to the symmetry energy has been studied in the past decade using the Boltzmann Uhling Uhlenbeck equation [16, 20]. One deficiency in the BUU model is the lack of cluster formation. Since clusters are emitted in experiment, it is important to understand the effect of clusters on neutron/proton ratios constructed in dynamical models. To understand this issue, we have performed simulations with the Improved Quantum Molecular Dynamics transport model [21, 22] using two equations of states which differ in the symmetry terms.

In the present study, we use the Improved Quantum Molecular Dynamics transport model code, ImQMD05 [21, 22] to study the multifragmentation processes. Within the ImQMD05 model, the mean field acting on nucleon wavepackets are derived from an energy functional where the potential energy U includes the full Skyrme potential energy with just the spin-orbit term omitted:

$$U = U_{\rho} + U_{md} + U_{coul} \qquad (1)$$

Here, U_{coul} is the Coulomb energy, while the nuclear contributions can be represented in local form with

$$U_{\rho,md} = \int u_{\rho,md} d^3 r \qquad (2)$$

and,

$$u_{\rho} = \frac{\alpha}{2}\frac{\rho^2}{\rho_0} + \frac{\beta}{\eta+1}\frac{\rho^{\eta+1}}{\rho_0^{\eta}} + \frac{g_{sur}}{2\rho_0}(\nabla\rho)^2 + \frac{g_{sur,iso}}{\rho_0}[\nabla(\rho_n - \rho_p)]^2 + \frac{C_s}{2}(\frac{\rho}{\rho_0})^{\gamma}\delta^2\rho + g_{\rho\tau}\frac{\rho^{8/3}}{\rho_0^{5/3}} \qquad (3)$$

where the asymmetry $\delta = (\rho_n - \rho_p)/(\rho_n + \rho_p)$, and ρ_n and ρ_p are the neutron and proton density, respectively. In the present work, the symmetry potential energy density of the form $\frac{C_s}{2}(\frac{\rho}{\rho_0})^{\gamma}\delta^2\rho$ is used in transport model comparisons. The energy density associated with the mean-field momentum dependence is represented as

$$u_{md} = \frac{1}{2\rho_0}\sum_{N_1,N_2=n,p}\frac{1}{16\pi^6}\int d^3p_1 d^3p_2 f_{N_1}(\vec{p}_1) f_{N_2}(\vec{p}_2) 1.57\left[\ln\left(1+5\times10^{-4}(\Delta p)^2\right)\right]^2 \qquad (4)$$

f_N are nucleon Wigner functions, $\Delta p = |\vec{p}_1 - \vec{p}_2|$, the energy is in MeV and momenta are in MeV/c. The resulting interaction between wavepackets is described in Ref. [23]. In this work, the coefficients used in above formulas are set to α=-356 MeV, β=303 MeV, η=7/6, g_{sur}=19.47 MeVfm^2, g_{suriso}=-11.35 MeVfm^2, C_s=35.19 MeV, $g_{\rho\tau}$ =0 MeV.

Isospin-dependent in-medium nucleon-nucleon scattering cross sections in the collision term and the Pauli blocking effects are described in [21,22, 24]. Clusters are constructed by means of the coalescence model widely used in QMD calculations in which particles with relative momenta smaller than P_0 and relative distances smaller than R_0 are coalesced into one cluster. In the present work, the values $R_0 = 3.5\,fm$ and $P_0 = 250 MeV/c$ are employed.

From the adopted assumptions, the symmetry energy per nucleon (in MeV) used in the simulations is a sum of kinetic and interaction terms.

$$E_{sym}(\rho)/A = \frac{1}{3}\frac{\hbar^2}{2m}\rho_0^{2/3}\left(\frac{3\pi^2}{2}\frac{\rho}{\rho_0}\right)^{2/3} + \frac{C_s}{2}\left(\frac{\rho}{\rho_0}\right)^{\gamma} \qquad (5)$$

where m is the nucleon mass. The density dependence of the symmetry energy used in the ImQMD model in the present data are plotted in Figure 1 as solid lines for γ=0.5, 1 and 2. For this particular parameterization, at density lower than the saturated density, the symmetry energy increase with decreasing γ while the opposite is true for supranormal density. Higher symmetry energy tends to drive the systems towards isospin equilibrations and that more neutrons will be emitted from a neutron rich system than compared to a neutron deficient system.

For comparison with experimental data from ref [15], we have performed calculations of collisions at impact parameter b=2 fm at incident energy of 50 MeV per nucleon for two systems: A=^{124}Sn+^{124}Sn and B=^{112}Sn+^{112}Sn. Central collisions at this incident energy are dominated by multifragmentation processes where a dilute system, at about 1/3 of the normal nuclear matter density is created. Figure 2 shows the singles ratios $R_{n/p}$(124) and $R_{n/p}$(112) as a function of the center of mass energy of the nucleons emitted between 70° and 110° in the center of mass frame in the right and left panel for the ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn reactions, respectively. The open symbols represent the singles ratios calculated using the weaker symmetry term with γ=0.5 and the solid symbols represent the singles ratios with γ=2. As expected, more neutrons are emitted from the more neutron rich ^{124}Sn+^{124}Sn system and by the systems with the weaker density dependence of the symmetry energy.

The shaded regions in the left panel of Figure 3 shows the range, determined by uncertainties in the simulations, of predicted double ratios DR(n/p)=$R_{n/p}$(124)/ $R_{n/p}$(112) as a function of the center of mass energy of the nucleons, obtained using two different density dependencies of the symmetry potential with γ=0.5 and 2. The double ratios $R_{n/p}$(124)/ $R_{n/p}$(112) are higher for the EOS with weaker dependence of symmetry energy on density, γ=0.5. For comparisons, the measured double ratio $R_{n/p}$(124)/ $R_{n/p}$(112) are plotted as solid stars. The general trend of the data with the double ratios increasing with the kinetic energy of the emitted nucleons is also reproduced. The data seem to be more consistent with predictions from calculations employing the EOS with γ=0.5. Unfortunately, the uncertainties in the measured $R_{n/p}$(124)/ $R_{n/p}$(112) are very large at high energy, $E_{C.M.}$>50 MeV, where the effects of sequential decays are expected to be suppressed. More accurate measurements are needed to distinguish the predictions between the two density dependence of the symmetry energy.

To examine the effect of sequential decays, we have simulated decays of fragments created in the collisions using the Gemini code [25]. The main effect of the sequential decays is the emission of more low energy neutrons than protons. While there are discernable effects on the ratios of low energy protons and neutrons, the effects largely cancel out in the double ratios. This underscores the advantage comparing double ratios of high energy neutrons and protons between predicted results and data. For future experiments, it is important to improve the accuracies of double ratio measurements for high energy neutrons and protons.

For models incapable of describing complex fragments formation, calculations can be compared to the coalescence invariant double ratios DR(n/p) constructed from measured spectra for light particles in the center of mass by including all neutrons and protons emitted at a given velocity, regardless of whether they are free or emitted within a light cluster, see right panel in Fig. 3. In such representation, fragments with Z>2 mainly contribute to the low energy spectra and do not affect the high energy data very much. For the lowest energy particles ($E_{C.M.}$/A<40 MeV), the coalescence invariant double ratios decrease by as much as a factor of two. Such change is also reflected in the data. For such comparisons, it is more important that high energy data be measured accurately.

The open diamond points in Fig. 3 are the IBUU04 [26] calculations with parameters that best describe the isospin diffusion data. For reference, the symmetry energy dependence on density used in IBUU04 is shown in Figure 1 as the dot-dashed line labeled x=-1. The latter Boltzmann Equation code includes more sophisticated treatment of the mean field including the momentum dependencies consistent with the Lane potential, and the in-medium nucleon-nucleon cross section are used. However, the predicted double ratios are too small compared to data, in spite of the inclusion of different effects. The difference between the two calculations could be due to the different treatment of the mean field and in-medium cross-sections.

Encouraged by the general agreement of the ImQMD model with data, we have extracted the calculated excitation function of the double ratios in Figure 4 for high energy neutrons and protons from incident energy of 35 to 150 MeV per nucleon. Since high energy particles are more robust in the construction of coalescence invariant quantities and less influenced by sequential decays, only high energy nucleons emitted between 70 and 110 in the center of mass with $E_{C.M.}$>40 MeV are included in the analysis. The magnitudes of the ratios decrease with increased incident energy independent of the strength of the symmetry term. In all cases, the double ratios are larger with weaker density dependence of the term. However, the differences between the two density dependence of the symmetry energy decreases with increasing beam energies. The excitation function provides another observable which may place more stringent constraints on the EOS of neutron matter as well as the other variables in the transport equation such as effective masses of neutron and protons and the isospin dependence of the in-medium cross-sections.

In summary, we have performed ImQMD transport equation simulations for the systems ^{124}Sn+^{124}Sn and ^{112}Sn+^{112}Sn. The obtained neutron to proton ratios are comparable in magnitude to the data. However, definitive determination of the equation of state requires more accurate measurement of the double neutron to proton ratios than the data currently available.

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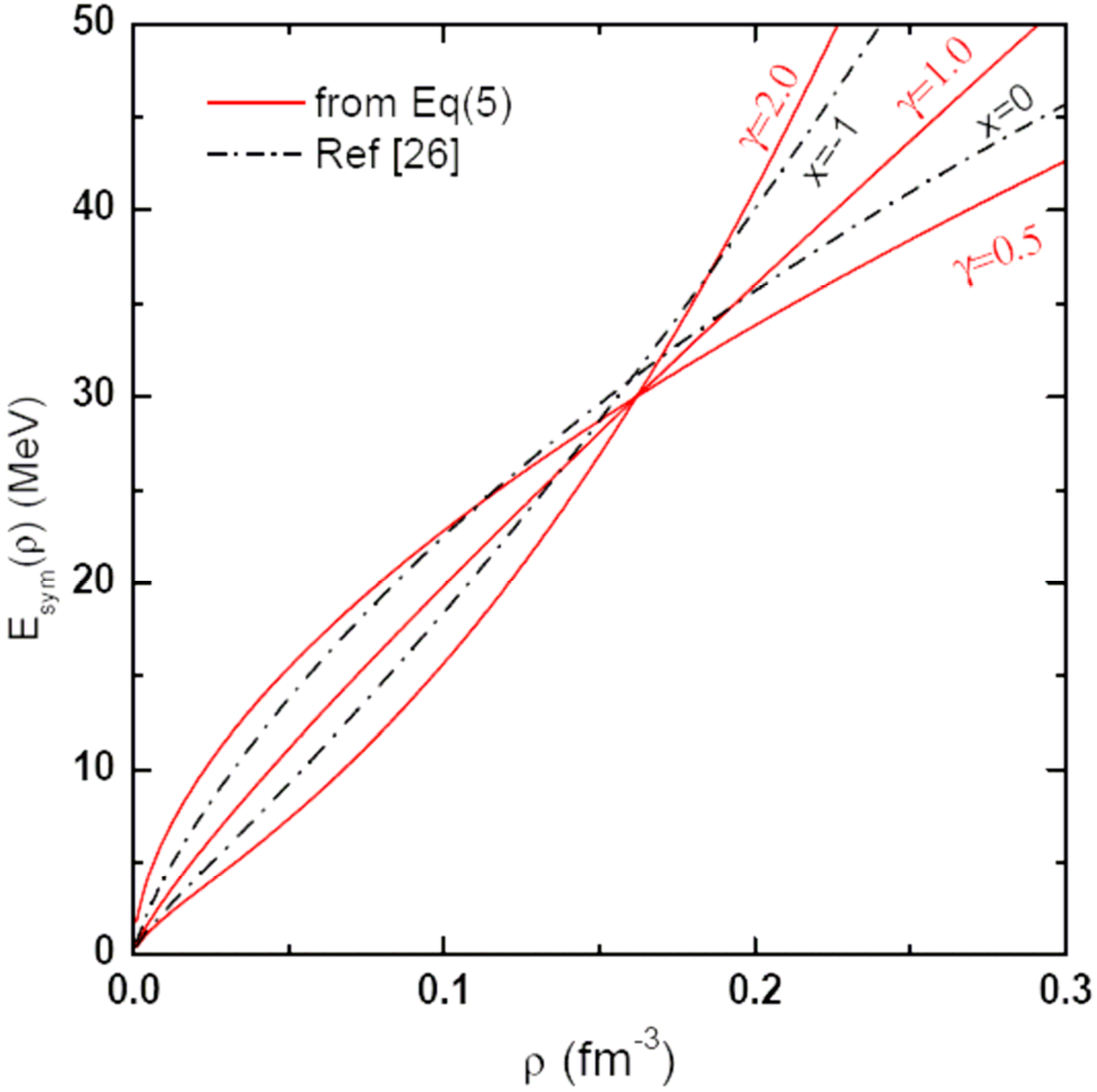

Fig.1 Symmetry energy per nucleon is plotted as a function of density for γ=0.5, 1.0 and 2 (solid lines) used in ImQMD simulations (Eq. 5). Dot-dashed lines labeled x=0, -1 are the corresponding density dependence of the symmetry energy from Ref [26].

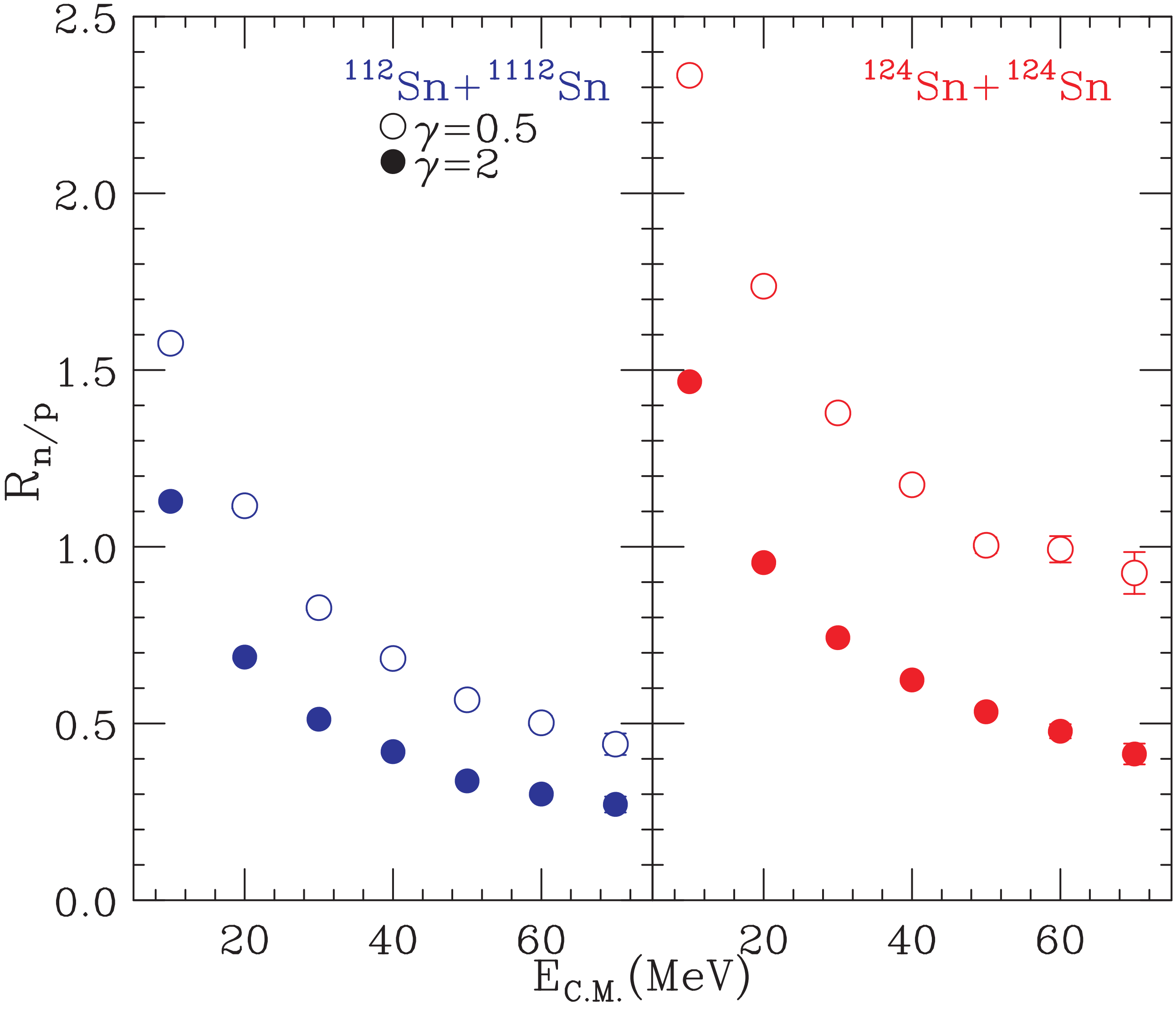

Fig.2: Ratio of neutron to proton yields for the ^{112}Sn+^{112}Sn reaction (left panel) and the ^{124}Sn+^{124}Sn reaction (right panel) as a function of kinetic energy for free nucleons emitted between 70° and 110° in the center of mass. Open symbols represent results from simulations using γ=0.5 in Eq. 5 and solid symbols correspond to results with γ=2.0.

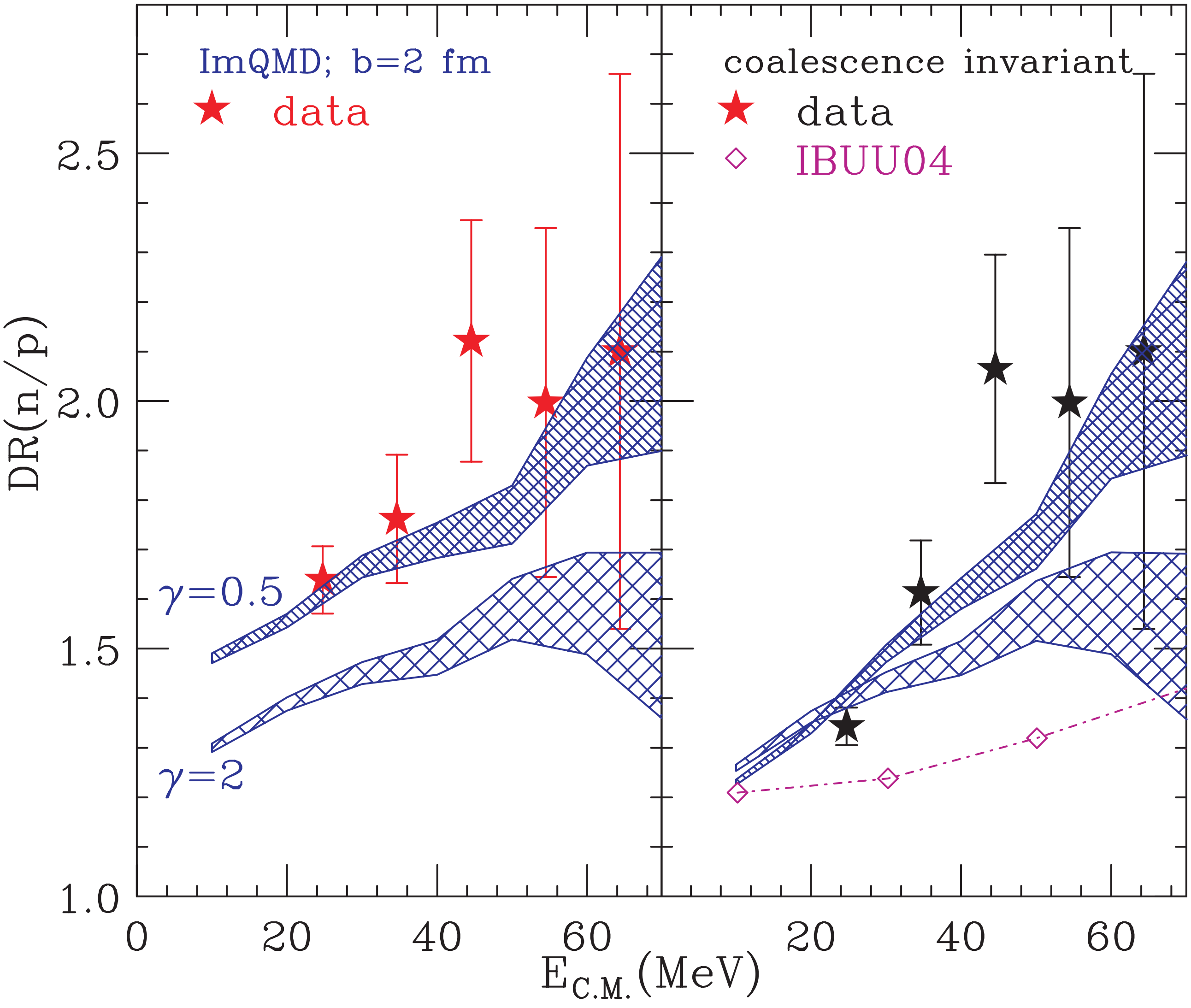

Fig.3 Free neutron proton double ratio (left panel), and coalescence invariant neutron proton double ratios (right panel) plotted as a function of kinetic energy of the nucleons. The shaded regions are calculations from the ImQMD simulations as discussed in the text. The open diamond points in right panel are the results from IBUU04 [26] calculations. The data (solid star points) are taken from Ref [15].

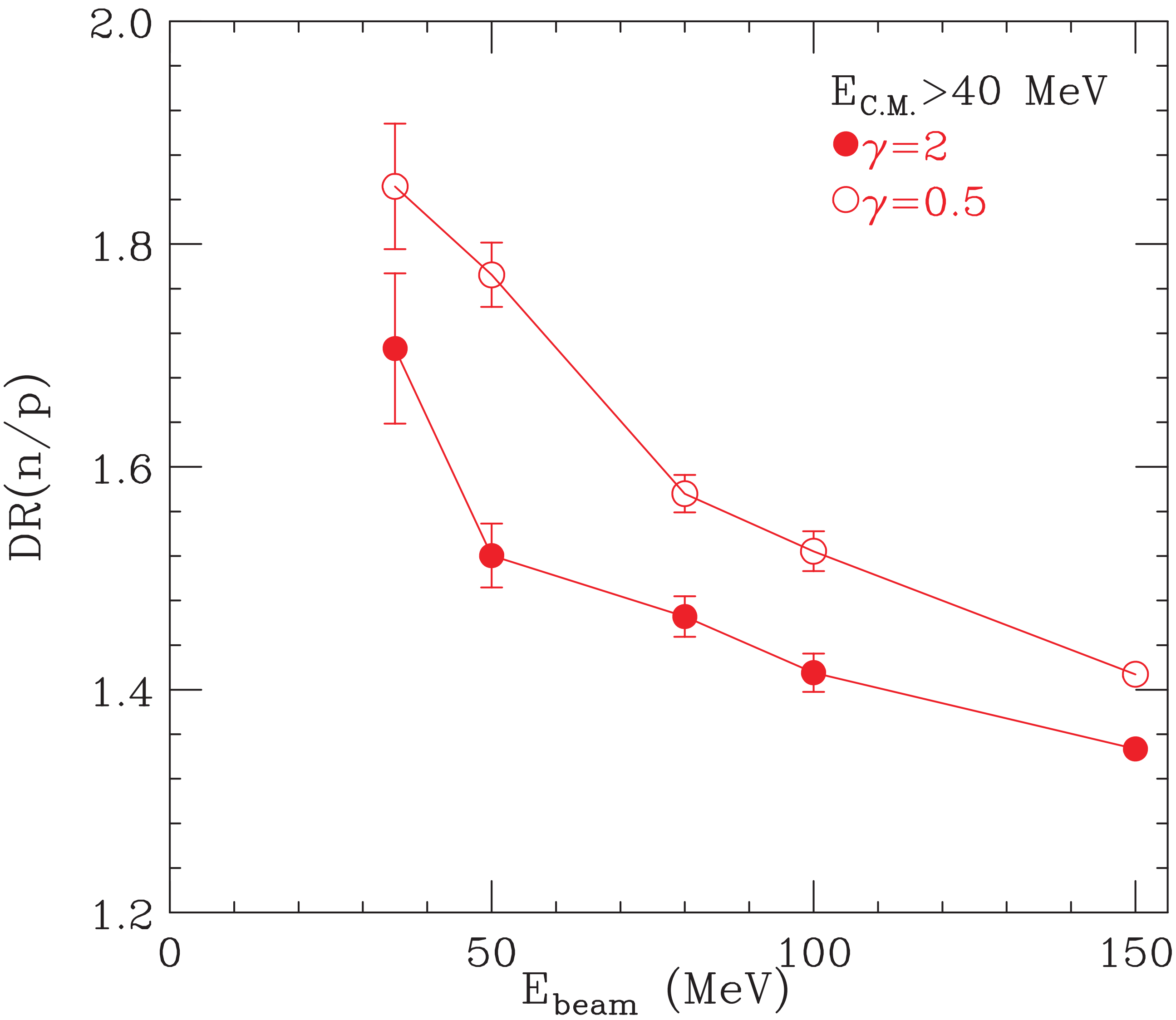

Fig.4, The excitation function of neutron to proton double ratios for γ=0.5 (open points) and γ=2.0(solid points) for high energy (E$_{C.M.}$>40MeV) neutrons and protons form incident energy of 35 to 150 MeV per nucleon.